Why Change Precalculus?

Rethinking Postsecondary Mathematics (March 2014)

Ohio Mathematics Initiative

On May 8, 2013, these conversations culminated in an Ohio Mathematics Summit, a meeting of mathematics faculty from all 36 USO campuses, which explored:

- policies that were impacting mathematics education in the state's two- and four-year postsecondary institutions;
- student retention issues confronting institutions across the state;
- concerns about the OTM's guidelines for mathematics, statistics and logic; and
- the effectiveness of quantitative pathways for STEM and non-STEM postsecondary majors.

Following the Summit, it was proposed that a steering committee of mathematics experts be formed to study national trends, current initiatives and available statewide and national data, and subsequently to make recommendations for future mathematics curricula in Ohio.

Issue 1.1

College algebra – the current gateway course in most mathematics departments -- is designed to prepare students for calculus and a subsequent series of mathematics courses. Yet, very few college algebra students intend to enroll or ever do enroll in a calculus course.

Research suggests that contextualizing mathematics promotes increased student engagement and improves completion rates. This is particularly important for students who expect to major in a mathematics-intensive discipline, but also has relevance for students who expect to major in the social sciences, business or other fields.

Recommendation 1.1: Improve student success in entry-level courses by aligning mathematics to academic programs of study and by improving instructional delivery mechanisms

USO institutions should begin by developing and offering entry-level mathematics courses, or by redesigning existing courses, to serve the needs of students in clusters of academic programs (e.g. the social sciences, business and finance, allied health and other STEM disciplines). In particular, departments should remove college algebra as the ***default*** mathematics course for non-STEM majors.

TMM002 – Precalculus: Ideas for Essential Learning Outcomes

Oho Transfer 36 (OT36)

To achieve this, the course (or sequence) should have the same architectural goals as Calculus. In a Precalculus course, students should:

- develop effective thinking and communication skills;
- operate at a high level of detail;
- state problems carefully, articulate assumptions, understand the importance of precise definition, and reason logically to conclusions;
- identify and model essential features of a complex situation, modify models as necessary for tractability, and draw useful conclusions;
- deduce general principles from particular instances;
- use and compare analytical, visual, and numerical perspectives in exploring mathematics;
- assess the correctness of solutions, create and explore examples, carry out mathematical experiments, and devise and test conjectures;
- recognize and make mathematically rigorous arguments;
- read mathematics with understanding;
- communicate mathematical ideas clearly and coherently both verbally and in writing to audiences of varying mathematical sophistication;
- approach mathematical problems with curiosity and creativity, persist in the face of difficulties, and work creatively and self-sufficiently with mathematics;
- learn to link applications and theory;
- learn to use technological tools; and
- develop mathematical independence and experience open-ended inquiry.

– Adapted from the MAA/CUPM 2015 Curriculum Guide

Decades Later, Problematic Role of Calculus as Gatekeeper to Opportunity Persists

Launch Years Initiative
Dane Center
August 9, 2021|By David Bressoud

The ubiquitous importance, yet inequitable availability, of calculus in high school has become a serious problem. Over 75% of students enrolled in a typical first calculus course in college have already taken it in high school. As a result, the remaining 25% of students who have never seen calculus before have to compete with students who are retaking a course they have already completed. Students who have not been exposed to the difficult concepts in calculus, who have not been introduced to the basic terminology or to the big ideas, are at a considerable disadvantage.

I have been amazed to discover that across the country it is typical that 25 or 30% of students who take their first calculus course in college fail. It seems to be a national expectation that a significant percentage of students will be lost—indeed, *should* be lost—from a STEM pathway after taking college calculus. Given those high course failure rates, it is not hard to see that the 25% of college calculus students who never took calculus in high school are at a tremendous disadvantage.

If the goal of taking calculus in high school is to pass out of Calculus I and enroll in the next course in the sequence, the data does not correspond:

- Only about one in five students who take calculus in high school actually skip over the first course in college.
- About 3 in 10 students who take calculus in high school will retake that course in college.
- About two-thirds of students who took calculus in high school and then retake Calculus I in college get an A or B in the college course.
- One in three students who took calculus in high school, however, get a C or lower, even though they are repeating a course in which they have already succeeded.
- Most disturbing of all, upward of 30 to 35% of students who successfully complete calculus in high school then go on to an assessment at the postsecondary level that places them into a precalculus course or a college algebra course. Some even wind up in remedial mathematics.

The existing high school course sequence, built to prepare students for calculus, remains problematic

A Common Vision

For Undergraduate Mathematical Sciences Programs in 2025

About the project

The *Common Vision* project is a joint effort, focused on modernizing undergraduate programs in the mathematical sciences, of the American Mathematical Association of Two-Year Colleges (AMATYC), the American Mathematical Society (AMS), the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM).

We began with an in-depth examination of seven curricular guides published by these five associations and spent a substantial amount of time identifying common themes in the guides. This report reflects a synthesis of these themes with our own research and input from project participants and other thought leaders in our community.

- *Beyond Crossroads* (AMATYC)
- *Guidelines for Assessment and Instruction in Statistics Education College Report (*ASA)
- *Curriculum Guidelines for Undergraduate Programs in Statistical Science* (ASA)
- *2015 CUPM Guide to Majors in the Mathematical Sciences* (MAA)
- *Partner Discipline Recommendations for Introductory College Mathematics and the Implications for College Algebra* (MAA)
- *Modeling across the Curriculum* (SIAM)
- *Undergraduate Programs in Applied Mathematics* (SIAM)

One of the most striking findings is that all seven guides emphasized this point, in particular:

The status quo is unacceptable.

What Is Rigor in Mathematics Rally?

Dana Center (2019)

UNDERSTANDING RIGOR IN THE CONTEXT OF AN EFFECTIVE MATHEMATICS COURSE

While rigor is important for mastery of mathematics, it is not a replacement for other elements essential to an effective course. We propose that one way to understand the role of rigor in an effective mathematics course is to imagine a rope2 with five interdependent and intertwined strands. The strands of the rope include:

Procedural fluency and skills – Using the definition offered by the National Research Council (2001), students have acquired procedural fluency when they have "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (p. 116). In arithmetic, for example, this means students are able to add, subtract, multiply, and divide numbers accurately and with confidence.

Conceptual understanding –Students demonstrate "comprehension of mathematical concepts, operations, and relations" (National Research Council, 2001, p. 116). Two examples illustrate this idea. In algebra, students show an understanding of when to use the quadratic equation, how to solve it, and how to interpret and use the results. In statistics, students show an understanding of when to use a particular inference test, state the assumptions, and demonstrate how to interpret and use the results.

Productive persistence –Students use tenacity in solving problems and employ a variety of effective learning strategies to successfully engage with coursework (Carnegie Foundation for the Advancement of Teaching, n.d.; Sylva & Whyte, 2013). For example, they are engaged in finding effective resources, motivated to wrestle with a problem until they can solve it, and interested in evaluating their work to find errors.

Application – Students correctly apply mathematical knowledge in new situations (National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010a). For example, application of algebra in calculus such as differentiating exponentials.

Rigor – Students use mathematical language to communicate effectively and to describe their work with clarity and precision. Students demonstrate that what they have done works, when it works, and why the procedure they selected is appropriate. The student can answer the question, "How do we know?"

Ohio Department of Education and Workforce

Strengthening Ohio High School Mathematics Pathways Initiative

What is Rigor in a Mathematics Classroom? (Ohio high school)

Students use mathematical language to communicate effectively and to describe their work with clarity and precision. Students demonstrate how, when and why their procedure works and why it is appropriate. Students can answer the question, “How do we know?”

A College Definition Would Look More Like:

Students use mathematical language to communicate effectively and to describe their reasoning with clarity and precision. Students explain how, when and why their reasoning appropriately accounts for all aspects of the situation. Students can answer the question, “How do we know, that we have accounted for everything?”

A Characterization of Calculus I Final Exams in U.S. Colleges and Universities

International Journal of Research in Undergraduate Mathematics Education (2016)

A review of student learning of ideas in introductory calculus (e.g., Carlson & Rasmussen, 2008) revealed that Calculus I students are generally not developing conceptual understanding of the central ideas of calculus. At the same time, the calculus reform movement has resulted in shifts in calculus curriculum to provide increased focus on student understanding of the course's key ideas (Ganter, 2001). With this increase in conceptual focus in the calculus curriculum, one might expect that calculus exams would include more items that assess students' understanding and ability to use the key concepts of calculus. Our analysis suggests this is indeed not the case.

Our examination of 150 contemporary Calculus I final exams revealed that these exams are primarily assessing students' ability to recall and apply procedures. There is little focus on assessing student understanding, with 85.21% of the 3,735 exam items being solvable by simply retrieving rote knowledge from memory, or recalling and applying a procedure. The Calculus I final exams rarely make use of real-world contexts, seldom elicit explanation or justification, and provide few opportunities for students to demonstrate or apply their understanding. Of the 150 exams we coded, 90% had 70% or more of the exams' items coded as "remember" or "recall and 45 apply procedure." In contrast, only 2.67% of the 150 exams had 40% or more of the items requiring students to demonstrate or apply understanding.

As a result of the low percentage of exam items at the "understand" or "apply understanding" levels of the item orientation taxonomy, we conclude that a large percentage of exam items failed to provide insight into how students understand the concepts on which their computational or procedural work is based. Hence, these results suggest that a large majority of Calculus I final exams being administered in colleges and universities in the United States encourage memorization of procedures for answering specific problem types and do not encourage students to understand or apply concepts of beginning calculus.

Our data further revealed that Calculus I final exams in U.S. colleges and universities have changed very little in the past 25 years relative to the percentage of exam items that require students to apply understanding of concept. Since final course exams commonly reflect the level of mastery and understanding students have attained at the end of the course, these data suggest that the calculus reform movement of the late 1980s has had little effect on what is being assessed in contemporary Calculus I courses in U.S. postsecondary institutions.

A Study of Students' Readiness to Learn Calculus

Springer International Publishing (2015)

Abstract The Calculus Concept Readiness (CCR) instrument assesses foundational understandings and reasoning abilities that have been documented to be essential for learning calculus. The CCR Taxonomy describes the understandings and reasoning abilities assessed by CCR. The CCR is a 25-item multiple-choice instrument that can be used as a placement test for entry into calculus and to assess the effectiveness of precalculus level instruction. Results from administering the CCR to first semester calculus students at the beginning of the semester revealed severe weaknesses in students' foundational knowledge and reasoning abilities for learning calculus. Correlating CCR results with course grades revealed that students with higher CCR scores are better prepared to succeed in beginning calculus. The CCR data further identified specific ways of thinking and concepts for which precalculus instruction could be improved to influence student learning and preparation for calculus.

Beginning Calculus Students are Not Prepared to Understand Calculus

An examination of the CCR response patterns of the 601 students completing CCR at the beginning of calculus revealed severe weaknesses in these calculus students' understandings and reasoning abilities of ideas on which calculus is built. The majority of students were unable to answer both proportional reasoning questions correctly and only 9 % of the 631 students answered all three function word problems (See Appendix A, Items 2, 7, 12), suggesting weakness in their ability to construct meaningful formulas by examining the quantities in a dynamic word problem context. Another area of difficulty was in students' ability to compose two functions.

A Cross-Sectional Investigation of the Development of the Function Concept

CBMS Issues in Mathematics Education (1998)

Conclusions and Implications

The following are general conclusions reached as a result of this research

- Function constructs develop slowly and their development appears to be facilitated by reflection and constructive activities. Interview results suggest that when students are confronted with engaging activities and provided time for reflection, student understanding is promoted. These results agree with those of Breidenbach et al. Their study found that understanding of functions was noticeably improved as a result of engaging students in constructive activities.
- Even the most talented students at the completion of college algebra still have many misconceptions. Analysis of the exam responses and interview transcripts for group 1 indicates that these students:
    - Do not understand the language of functions, in particular
        - What it means for one quantity to be a function of another
        - The role of parentheses in the function representation
        - That the "function value" is referring to the y-value. (conventional labeling)
    - Do not know how to represent real world functional relationships using algebraic or graphical function representations.
    - Do not make a distinction between zeros of functions and solutions of equations.
    - Do not effectively interpret dynamic graphical information.
    - Do not understand the general nature of a function.
    - Do not understand the role of the independent and dependent variables in an algebraic function representation.
    - Do not represent graphically, covariant aspects of a real-world situation.
    - Do not possess helpful mathematical habits.

School students’ preparation for calculus in the United States

FIZ Karlsruhe 2021

Abstract

Researchers have been interested in students’ transition to calculus since the early 1900s. One line of inquiry highlights students’ understandings of high school mathematics as impeding or supporting their successful transition to university mathematics. This paper addresses an underlying question in this line of inquiry: does school mathematics provide opportunities for students to develop productive meanings for calculus? This article reports on U.S. calculus students’ responses to items that assessed students’ variational reasoning, meanings for average rate of change, and representational use of notation—ideas ostensibly addressed in school mathematics. To make sense of students’ difficulty on these items we sought to understand the opportunities students had to reason with these ideas prior to calculus. We use two data sources to understand the likelihood that students have opportunities to construct productive meanings for function notation, variation, and average rate of change in their secondary mathematics education: meanings for these ideas supported by precalculus textbooks and meanings secondary teachers demonstrated. Our analysis revealed a disconnect between meanings productive for learning calculus and the meanings conveyed by textbooks and held by U.S. high school teachers. We include a comparison of meanings held by U.S. and Korean teachers to highlight that these meanings are culturally embedded in the U.S. educational system.

Conclusion

The analyses presented here suggest that U.S. students have limited opportunities to construct mathematical meanings productive for understanding calculus. We argue there is evidence of a large disconnect between meanings conveyed by textbooks and held by teachers and meanings that would be productive for students’ understanding of major ideas in calculus.